Velocity and Temperature Spectra in the Convective Atmospheric Surface Layer – Multipoint Monin-Obukhov Similarity: Theory and Field Measurements

Kirill Barskov [1] Shane Mayor [2] Pierre Dérian [2] Chenning Tong [1]

[1]Clemson University [2]California State University, Chico

Multipoint Monin-Obukhov Similarity Theory (MMO)

The Monin-Obukhov Similarity Theory is the theoretical foundation for understanding the atmospheric surface layer (Monin and Obukhov 1954). It can scale the mean profiles and the magnitudes of the inertial-range spectra (not the scaling exponents).

However, it has been known in the late 1950s that it cannot scale spectra for scales larger than the distance from the surface ($k < 1/z$) (Kaimal et al. 1972, Kaimal 1978, and Caughey and Palmer 1979)

The Multipoint Monin-Obukhov Similarity Theory (MMO) resolves this deficiency (Tong and Nguyen 2015, Tong and Ding 2018).

It predicts that non-dimensional multipoint statistics in the surface layer to be functions of z/L and kL. For example, the velocity spectra have the form

$$\phi(k_h L, z/L), \quad \text{for } kz < 1.$$

For $-z/L < 1$, it further predicts two scaling ranges, the convective and dynamic ranges.

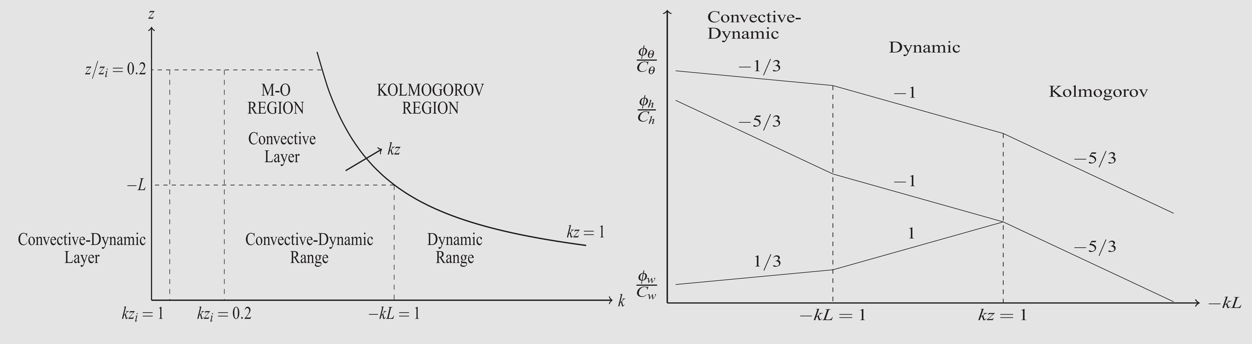


Figure 1. Left: Schematic of the scaling regions in the convective atmospheric surface layer. Right: Schematic of the predicted scaling of the temperature spectrum, normalized by $C_\theta = (\kappa\beta)^{-2/3} Q^{4/3} (-L)^{1/3}$, and the horizontal and vertical velocity spectra, normalized by $C_h = C_w = (\kappa\beta Q)^{2/3} (-L)^{5/3}$.

Ring-integrated spectra vs. one-dimensional spectra

One-dimensioal spectra, ubiquitous in micrometeorology, are obtained as

$$\phi_1(k_1) = \int_{k_{h1}=k_1} \phi(\mathbf{k_h})\, d\mathbf{k_h}.$$

They therefore contain aliasing effects, do not represent the variances contained at each scale, and are not suitable for testing spectral prediction of MMO.

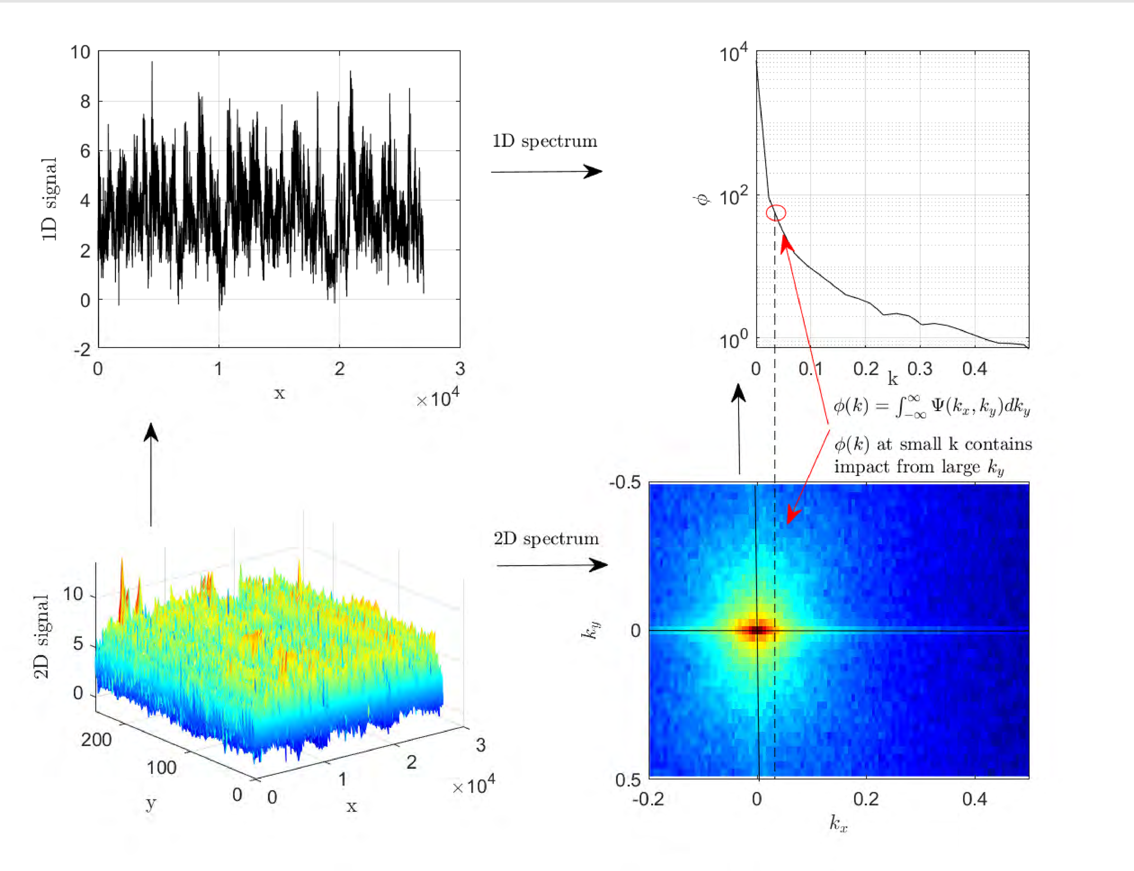


Figure 2. Relationship between 1D- and 2D-spectra

To overcome the issue of aliasing, spectra integrated over each wavenumber magnitude, the ring-integrated spectra,

$$\phi'(k) = \int_{k_h=k} \phi(\mathbf{k_h})\, d\mathbf{k_h}$$

are needed.

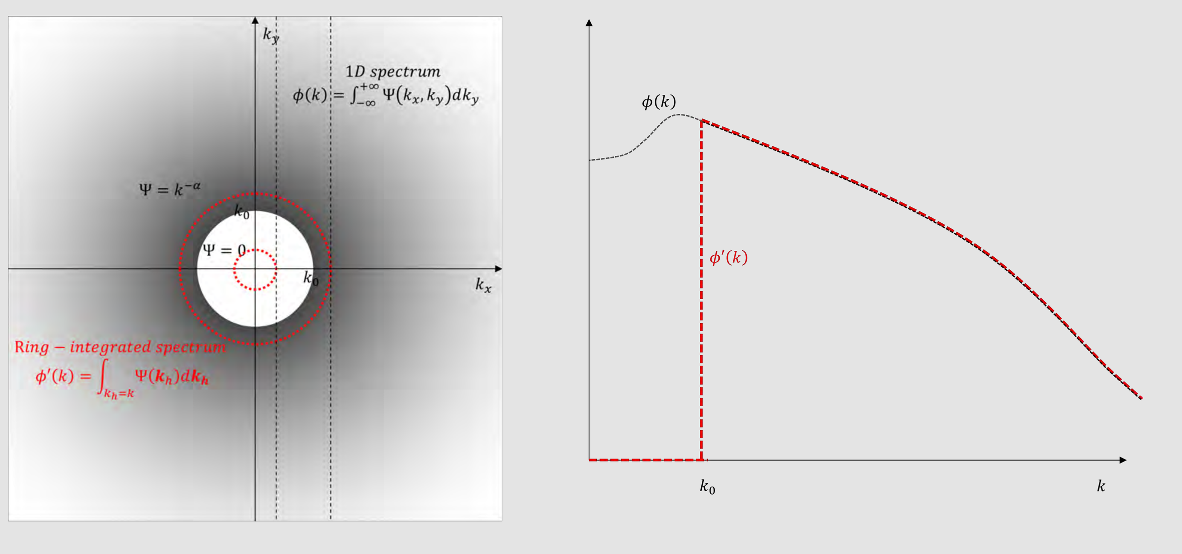
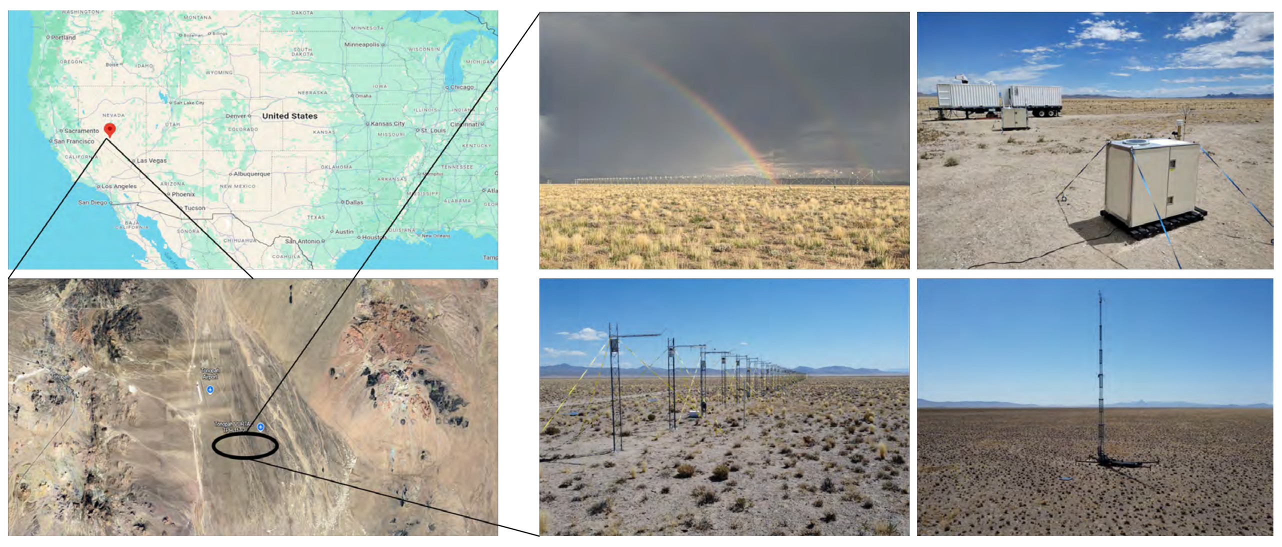


Figure 3. Demonstration of failing 1D spectra to describe variances for small scales. If $\Psi = 0$ for $|k| < k_0$ 1D spectrum will still have non-zero values in these scales.

Multi-point Monin-Obukhov similarity Horizontal Array Turbulence Study (M^{2}HATS)

23 July 2023 - 24 September 2023. Tonopah, Nevada, USA



To fully validate the new theory, a field measurement campaign was conducted, in which a large suite of in situ and remote sensing instruments were deployed.

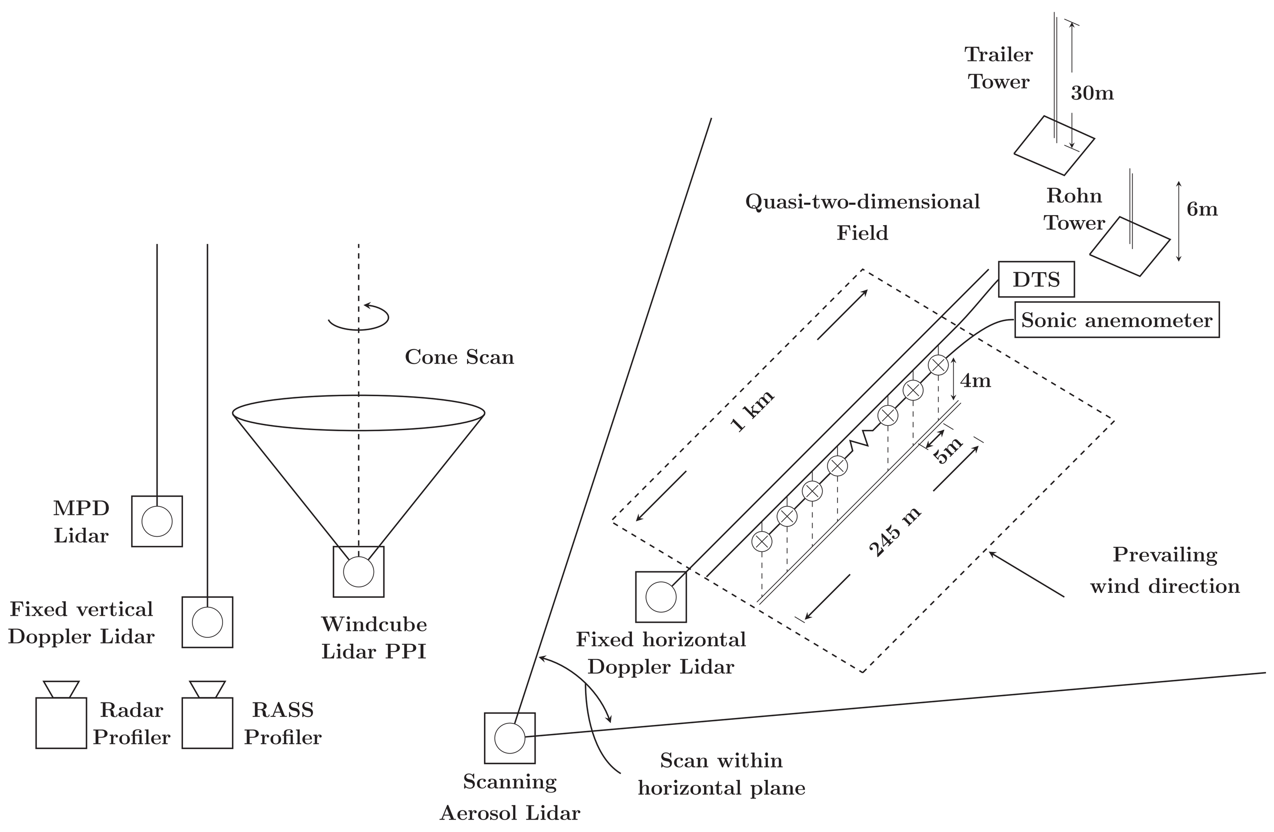


Figure 4. Schematic of the measurement set-up.

- **Horizontal anemometer array:** Campbell Scientific CSAT 3,3A and 3B, sample rates are 20,30,50,60 Hz. 50x4.5m height towers with 5m spacing. Quasi-two dimensional turbulent velocity fields. Turbulence spectra in scales 5-245 m. Every third tower: H2O/CO2 gas analyzer (Campbell Scientific IRGA EC150), Temperature and RH (Sensirion SHT85), Nanobarometer (Paroscientific 6000).
- **Horizontal Doppler lidar:** Metek Halo Lidar. Beam is along the sonic array. Turbulence cross-wind velocity component spectra in scales 30m - 3 km.
- **The Raman-shifted Eye-safe aerosol lidar (REAL):** Two-dimensional horizontal velocity fields from high resolution images of the aerosol distribution using Typhoon wavelet-based optical flow software; Turbulence horizontal velocity spectra at 5m height in scales 100 m - 3 km.
- **Distributed temperature sensors (DTS):** Fiber optic system. Quasi-two dimensional temperature field. Temperature spectra at 2.3m height in scales 2-1000 m.
- **Multi-level towers:** 8 levels (0.62m, 1.17m, 2.11m, 3.02m, 4.2m, 6.89m, 15.45m, 28.55m), 3D sonic anemometers (CSAT3A), H2O/CO2 gas analyzer (Campbell Scientific IRGA EC150), Nanobarometer (Paroscientific 6000)
- **Micro-Pulse DIAL lidar(MPD):** Temperature and water vapor profiles (300 m -5 km height).
- **Vaisala Windcube 200S Lidar (VAD scans):** One-minute 360° PPI scans generating VAD winds from 50 m up to approximately 2km.
- **UVA Halo lidar (vertically pointing):** Constant stare vertically, 1 s, 30 m resolution, typical range 50 m to approximately 2 km.

Data processing

In total 44 statistically stationary time periods ranging from 45 minutes to 4 hours (45 hours 34 minutes in total) were selected. The mean wind direction was approximately perpendicular to the sonic array and the mean wind speed was more than 2 m/s for all the periods, which allow to apply Taylor's hypothesis to obtain quasi-two-dimensional fields for sonic array, horizontal lidar and DTS data. Then 2D spectra were calculated and ring-integration was conducted.

The u, v and w -spectra were obtained using the sonic array.

The v -spectrum was obtained using the horizontal Doppler lidar. Attenuation of the Doppler lidar spectra due to spatial averaging (30 m spatial resolution) along the beam and time averaging (over several integration periods) have been corrected by dividing the (sinc) transfer function of the averaging filter.

The u and v-spectra were obtained using REAL. REAL frames were cross-checked with horizontal Doppler lidar data to select only relevant scanning frames. The two-component velocity fields obtained using REAL/Typhoon had an effective velocity spatial resolution of approximately 100 m; therefore, wavenumbers larger than $k \approx 0.03$ m^{-1} were not included to analysis.

θ - spectrum was obtained using DTS.

Results

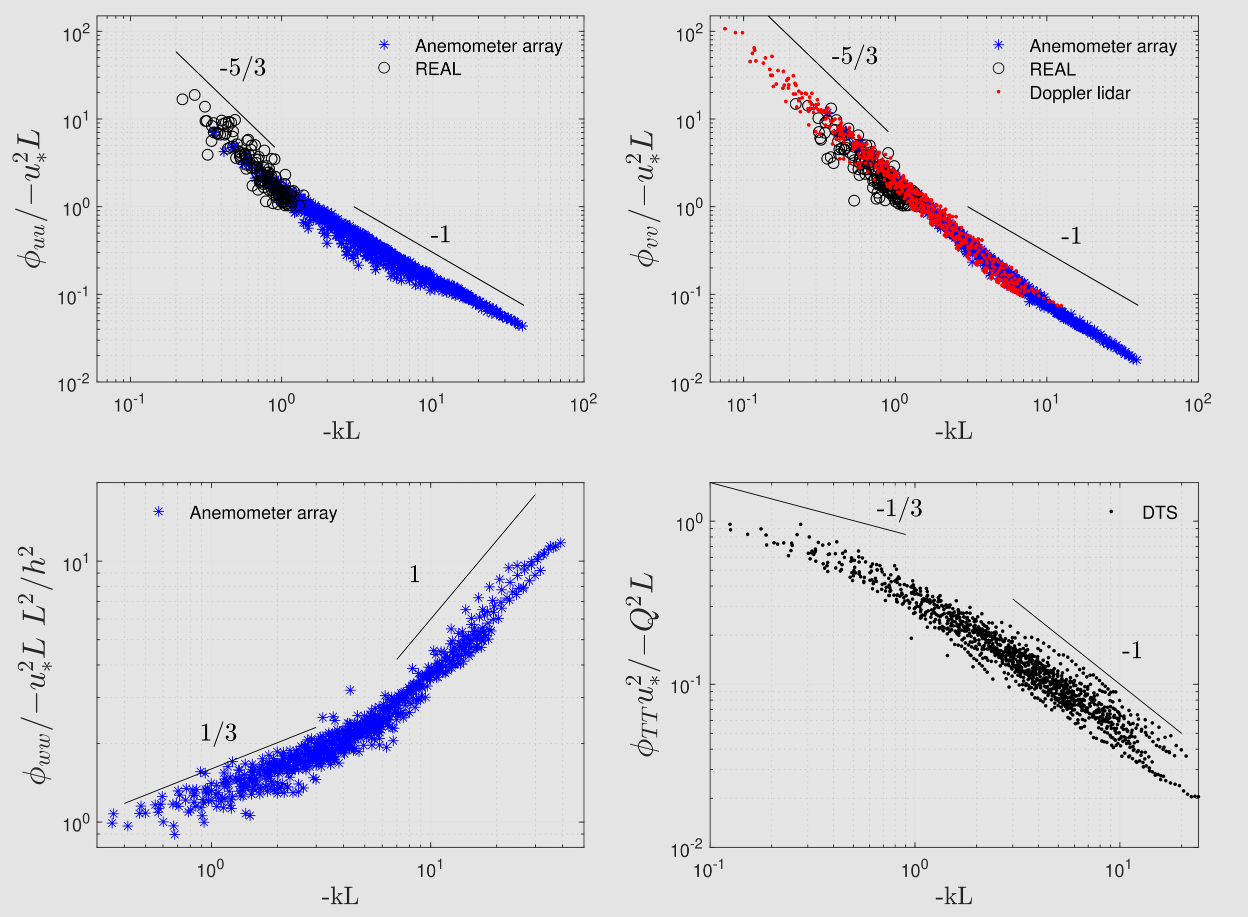


Figure 5. Normalized integrated spectra. Results from different instruments are consistent. Leading order terms follow to MMO prediction

The measured spectra support the MMO prediction.

The u-spectrum ϕ_{uu} is consistent with the MMO scaling of $k^{-5/3}$ for the convective range ($-kL < 1$) and k^{-1} for the dynamics range ($-kL > 1$ and $kz < 1$), whereas ϕ_{vv} shows a k^{-1} range only for approximately $-kL > 7$. We do not yet have a good explanation for this behavior of the v-spectrum. We note that both previous studies and present work have found that for one-dimensional spectrum, the v-spectrum (frequency spectrum) appears to show a k^{-1} scaling range, whereas the u-spectrum does not. This issue needs further investigation.

The w-spectrum shows the MMO scaling of $k^{1/3}$ and k^{1} for the convective and dynamic ranges, respectively. The transition wavenumber is approximately $-kL = 5$, between those of the u- and v-spectra, consistent with the kinematic relationship between these spectra imposed by continuity. Note that the one-dimensional w-spectrum cannot roll off at small wavenumbers, demonstrating the importance of measuring ring-integrated spectra.

The θ-spectrum shows the MMO scaling of $k^{-1/3}$ and k^{-1} for the convective and dynamic ranges respectively. The transition is approximately at $-kL = 1$.

Conclusions

- The M2HATS field campaign deployed a wide range of in situ and remote sensing instruments, providing unprecedented comprehensive data for testing MMO, including the spectra.
- The results follow MMO for the leading order terms for the ring-integrated velocity and temperature spectra.
- The M^{2}HATS campaign will make data available to the atmospheric community as well as the wider scientific community.

Acknowledgment

We would like to express our gratitude to the Nye County, Nevada, Board of Commissioners and to Mark Peterson at the Tonopah Airport FBO for their generous support before and during the campaign.

We would also like to sincerely thank the NCAR Earth Observing Laboratory staff for their professionalism and willingness to always provide extra support. They were also fantastic to work with, making the time in the field much more enjoyable.

We would also like to acknowledge the Physical and Dynamic Meteorology Program at the National Science Foundation for their support through grants AGS-2054983 and AGS-2054969. We appreciate their understanding and support while we navigated issues associated with changing field sites at the last minute and delays due to the COVID-19 pandemic.

References

Monin, A. S. and A. M. Obukhov (1954). "Basic laws of turbulent mixing in the ground layer of the atmosphere". In: *Trans. Inst. Teoret. Geofiz. Akad. Nauk SSSR* 151, pp. 163–187.

Kaimal, J. C. et al. (1972). "Spectral characteristic of surface-layer turbulence". In: *Q.J.R. Met. Soc.* 98, pp. 563–589.

Kaimal, J. C. (1978). "Horizontal velocity spectra in an unstable surface layer". In: *J. Atmos. Sci.* 35, pp. 18–24.

Caughey, S. J. and S. G. Palmer (1979). "Some aspects of turbulence structure through the depth of the convective boundary layer". In: *Q.J.R. Met. Soc.* 105, pp. 811–827.

Tong, C. and K. X. Nguyen (2015). "Multipoint Monin-Obukhov Similarity and Its Application to Turbulence Spectra in the Convective Atmospheric Surface Layer". In: *J. Atmos. Sci.* 72, pp. 4337–4348.

Tong, C. and M. Ding (2018). "Monin–Obukhov Similarity and Local-Free-Convection Scaling in the Atmospheric Boundary Layer Using Matched Asymptotic Expansions". In: *J. Atmos. Sci.* 75, pp. 3691–3701.